Book Review

Z. Naniewicz and P. D. Panagiotopoulos: *Mathematical Theory of Hemivariational Inequalities and Applications.* Marcel Dekker, Inc. 1995. XVI, 267 pp., $120. ISBN 0-8247-9330-7 (Monographs and Textbooks in Pure and Applied Mathematics 188).

Nonsmooth nonconvex energy functions appear frequently in the mechanical and engineering problems as well as in the economy and biology and provide many fascinating aspects especially for mathematicians. There is indeed a growing interest in such problems during the last years because the new tools of nonsmooth analysis permit the further study of them with important results. Nonsmooth and nonconvex energy functions are treated by means of some generalized differential calculi, where the calculus of F. H. Clarke plays an important role, and they give rise to a new type of variational expressions called hemivariational inequalities. In the book of Z. Naniewicz and P. D. Panagiotopoulos, several types of hemivariational inequalities are studied concerning the existence of their solutions. The arising problems are characterized by nonsmoothness and by the lack of convexity, facts which make their mathematical treatment fascinating. The book contains many original results and the developed mathematical theories permit the study of until now unsolved boundary value problems in mechanics, engineering and economics.

The book is directed to mathematicians, primarily working in nonlinear and nonsmooth analysis and its applications, but also to researchers in mechanics, engineering and economics. The reader is expected to have a good knowledge of the basic functional analysis.

The first chapter gives a comprehensive review of the fundamental tools needed in the next chapters, i.e., convex and nonconvex nonsmooth analysis, elements of the theory of maximal monotone operators and of variational inequalities. Moreover, a part of it is devoted to some applications leading to hemivariational inequalities. In Chapter 2 some elements of the theory of pseudo-monotone and generalized pseudo-monotone multivalued operators are given. These notions are used in the sequel for the study of locally Lipschitz energy functionals and indicator energy functionals of nonconvex closed sets having generalized gradients with these properties. Finally the notion of quasi-pseudomonotonity is introduced and corresponding propositions are proved.

Chapter 3 deals with a type of hemivariational inequality wich was the first which has been studied. They arise in the case of one-dimensional nonconvex, nonsmooth energy functions, i.e., in the case of one-dimensional, nonmonotone, multivalued nonlinearities. First the coercive case is treated and subsequently an existence result for the more complicated semicoercive case is given. Then a more general type of hemivariational inequalites, the so called variational-hemivariational inequalities are studied and some existence and approximation results are proved. Finally the relation of hemivariational inequalities to the corresponding substationarity problems is investigated.

In Chapter 4 a more general multidimensional type of hemivariational inequalities is studied. First two classes of locally Lipschitz functions are introduced and their properties are studied. The first class, consists of functions whose generalized gradient is quasi-pseudomonotone, while the second of functions with pseudo-monotone generalized gradients. The theory of multivalued pseudomonotone operators permits the derivation of important existence results for hemivariational inequalities involving functions from these two classes. Variational-hemivariational inequalities and quasi-hemivariational inequalities are then studied and certain interesting existence results are obtained.

Chapter 5 is devoted to the study of hemivariational inequalities and variational-hemivariational inequalities on vector-valued function spaces, on the assumption that certain directional growth conditions hold.

Chapter 6 concerns noncoercive hamivariational inequalities arising in the study of free boundary problems. The authors study as a pilot problem a system consisting of a variational and a hemivariational inequality related to the continuous delamination model.

In Chapter 7 the theory of constrained problems for nonconvex star-shaped admissible sets in developed. Making use of hemivariational inequalities the authors prove the existence of solutions of variational inequalities on closed star shaped admissible sets.

The abstract results of the previous Chapters are illustrated by many applications in mechanics, engineering and economics. All the applications concern problems which involve nonsmooth, nonconvex energy functions and which are treated mathematically for a first time here. This is made possible by the methods developed in the book. The applications concern nonmonotone contact law problems, friction laws with nonconvex superpotentials, nonconvex plasticity, adhesively connected layered plates, delamination effects, viscoplastic flows, masonry structures, semipermeability problems and nonlinear network flow problems among others.

The contents of the book are clearly presented and the proofs are carefully explained. The book contains many original results and is recommended to all readers interested in nonsmooth problems in mathematics, mechanics, engineering and economics.

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